ESD-TR-78-395

LEVEL II

(12)

Technical Note 1978-46

R. E. Stovall

A Gaussian Noise Analysis of the "Pseudo-Coherent Discriminant"

29 December 1978

Prepared for the Defense Advanced Research Projects Agency
under Electronic Systems Division Contract F19628-78-C-0002 by

Lincoln Laboratory
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LEXINGTON, MASSACHUSETTS

Approved for public release; distribution unlimited.

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology. This work was sponsored by the Defense Advanced Research Projects Agency under Air Force Contract F19628-78-C-0002 (ARPA Order 2752).

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

Raymond L. Loiselle

Raymond L. Loiselle, Lt. Col., USAF
Chief, ESD Lincoln Laboratory Project Office

12

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

6

A GAUSSIAN NOISE ANALYSIS OF THE 'PSEUDO-COHERENT DISCRIMINANT'

10 Robert E. STOVALL

Group 45

9 TECHNICAL NOTE 1978-46

14 TN-1978-46

11 29 DEC[redacted]78

12 26p.

15 F19628-78-C-0002, ARPA Order-2752

Approved for public release; distribution unlimited.

18 ESD

19 TR-78-395

LEXINGTON MASSACHUSETTS

207 650

ABSTRACT

The "Pseudo-Coherent Discriminant" for radar detection of fixed targets in ground clutter is investigated. Performance of the technique in Gaussian noise interference is analyzed and compared with Marcum's results for square law detection of a steady signal.

CONTENTS

A GAUSSIAN NOISE ANALYSIS OF THE "PSEUDO-COHERENT DISCRIMINANT"

INTRODUCTION

Renewed interest in polarization techniques for radar detection of fixed targets in clutter has developed as a consequence of test results reported for the so-called "Pseudo-Coherent Discriminant (PCD)," (also referred to as "Polarimetric Processing" in other circles). In this brief note we shall describe the technique and present a first order analysis in an effort to understand its potential. This first analysis uses Gaussian noise as the interference statistic rather than one of the conventional clutter statistics. While performance of PCD in the presence of noise cannot be extrapolated exactly for the clutter problem, it is felt it can provide a good indication. In addition, by comparing performance against the well-known results obtained by Marcum for square law detection of a steady target in noise, PCD can be measured against an established benchmark.

TECHNIQUE DESCRIPTION

Briefly, PCD is a technique which discriminates targets on the basis of the relative phase between simultaneously received horizontal and vertical polarization channels for a transmitted circularly polarized waveform. Each channel is hard limited to remove amplitude information, and relative phase between the two is measured (or more exactly, the sine of the relative phase) using one channel as the "coherent" local oscillator, hence the designation "pseudo-coherent".

The basis for this approach relies on a target model consisting of flat plate and corner reflector, both dihedral and trihedral, scattering elements. Each of these target scatterers returns either the same or opposite sense circular polarization with the relative phase between simultaneously received horizontal and vertical polarizations being either $\pm\pi/2$. Single pulse clutter return, on the other hand, is postulated to have a relative phase which is nearly uniformly distributed over the interval $-\pi$ to π. Figure 1 shows the single pulse probability density functions for ϕ_{rel} for a target in noise (S/N = 3 dB) and for noise alone. Figure 2 shows the probability densities for $\sin \phi_{rel}$.

Detection cannot be accomplished on the basis of one pulse or even multiple pulses of the same frequency. Using pulse-to-pulse frequency agility and integration of the relative phase (or $\sin \phi_{rel}$), the clutter probability density approaches a Gaussian density function centered at $\phi_{rel} = 0$. If the relative phase of the target return is centered near $\phi_{rel} = \pi/2$, then integration will yield an N-pulse probability density centered at $\pi/2$. A threshold may then be set to discriminate between these two N-pulse density functions. Figure 3 shows the probability densities resulting from integration of 4 pulses.

NOISE ANALYSIS

For this analysis of PCD, the interference statistic chosen was Gaussian noise. Two considerations dictated the choice. First, noise has ideal clutter characteristics since the probability density for ϕ_{rel} is uniformly distributed from $-\pi$ to π. As a result, this should be considered a best case analyis. Second, performance can be compared against the well-known results of Marcum for a steady signal in noise.

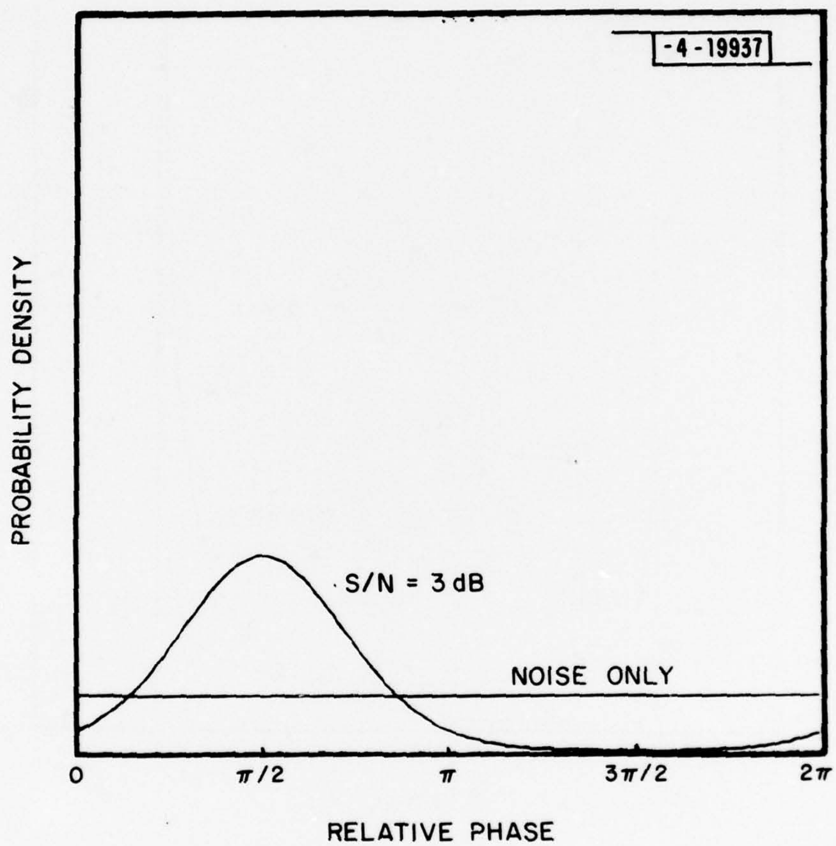

Fig. 1. Probability density for relative phase, ϕ_{rel}.

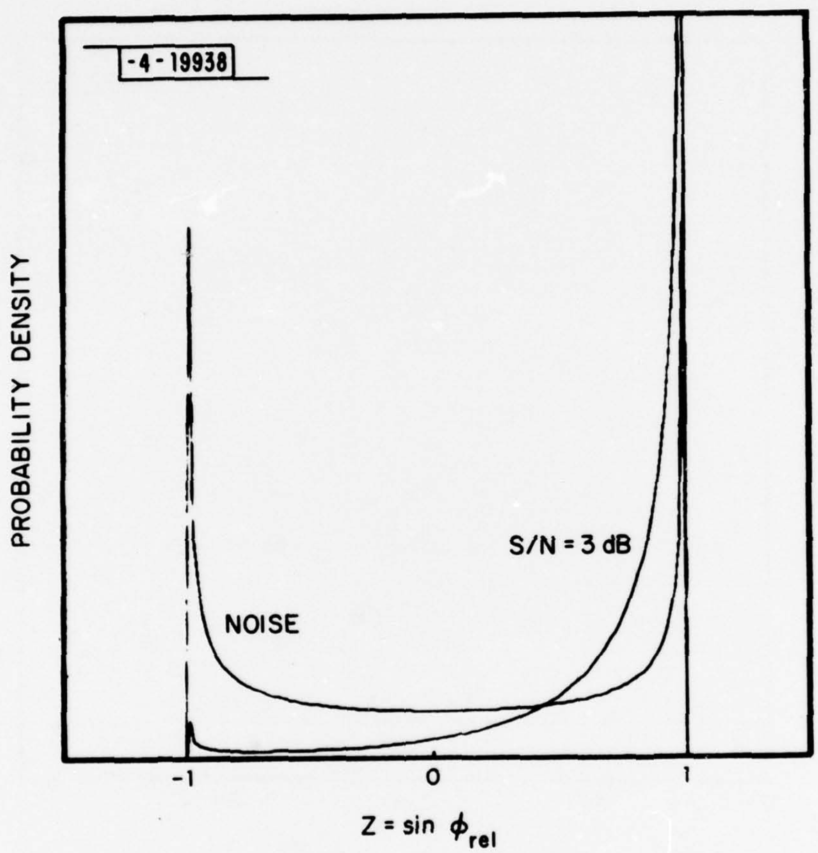

Fig. 2. Probability density for $Z = \sin \phi_{rel}$.

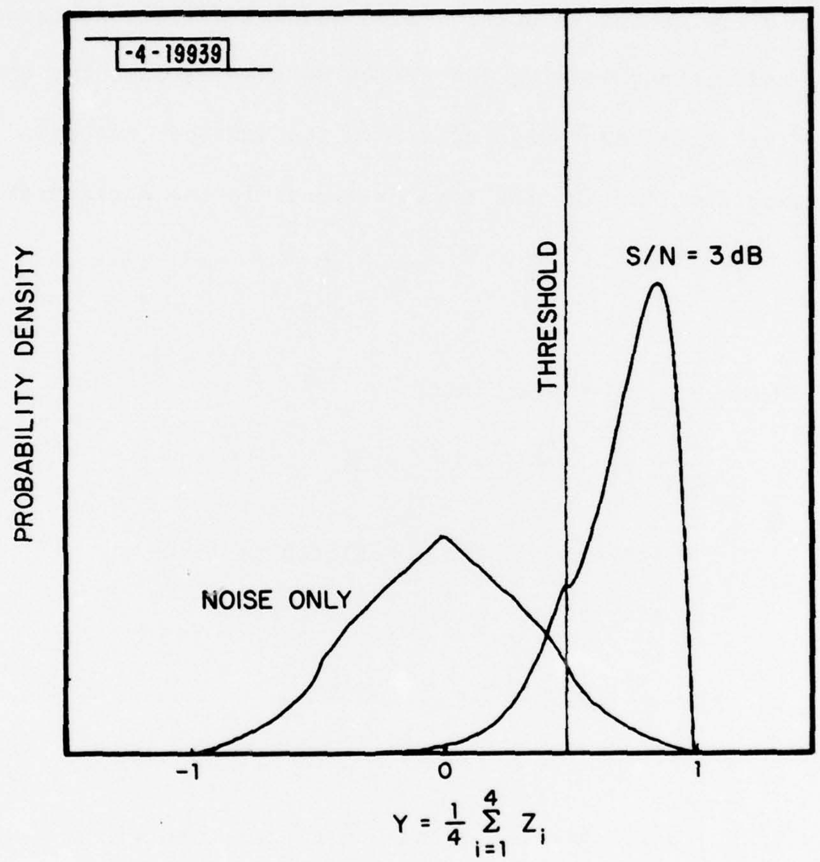

Fig. 3. Probability density for 4-pulse integration of $Z = \sin \phi_{rel}$.

Let X_H and Y_H represent the real and imaginary parts of the horizontal polarization channel, and X_V and Y_V the real and imaginary parts of the vertical polarization channel. The return shall be assumed that of a corner reflector in noise. Each channel will be synchronously demodulated, and without loss of generality we shall assume the constant corner reflector return has phase of $\pi/2$ in the vertical channel. The probability density function for the complex signal in the horizontal channel is thus

$$p(X_H,Y_H)dX_H dY_H = p(X,Y)dXdY$$

$$= \frac{1}{2\pi\alpha^2} \exp\{\frac{-(X-C)^2 - Y^2}{2\alpha^2}\}dXdY$$

let X/α, Y/α be normalized amplitudes,

and let $X = r\cos\theta$, $Y = r\sin\theta$

$$\therefore p(r,\theta) = \frac{1}{2\pi} \exp\{\frac{-r^2 - C^2 + 2Cr\cos\theta}{2}\}r$$

$$p(\theta) = \int_0^\infty p(r,\theta)dr = \frac{1}{2\pi}\int_0^\infty \exp\{-\frac{1}{2}[r^2-2Cr\cos\theta + C^2]\}rdr$$

$$= \frac{1}{2\pi} \exp\{-\frac{1}{2} C^2(1-\cos^2\theta)\} \int_0^\infty \exp\{-\frac{1}{2}(r-C\cos\theta)^2\}rdr$$

$$\text{let } \zeta = r - C\cos\theta \Rightarrow d\zeta = dr$$

$$= \frac{1}{2\pi}\exp\{-\frac{C^2}{2}\sin\theta\}\int_{-C\cos\theta}^{\infty}\exp\{-\frac{\zeta^2}{2}\}(\zeta + C\cos\theta)d\zeta$$

$$= K\int_{-C\cos\theta}^{\infty} e^{-\zeta^2/2}\zeta d\zeta + K\,C\cos\theta\int_{-C\cos\theta}^{\infty} e^{-\zeta^2/2}d\zeta$$

$$\text{let}\quad t = \zeta^2/2 \quad\text{and}\quad s = \zeta/\sqrt{2}$$

$$dt = \zeta d\zeta \qquad ds = \frac{1}{\sqrt{2}}d\zeta$$

$$K = \frac{1}{2\pi}\exp\{-\frac{C^2}{2}\sin^2\theta\}$$

$$= K\int_{\frac{C^2\cos^2\theta}{2}}^{\infty} e^{-t}dt + K\sqrt{2}\,C\cos\theta\int_{\frac{-C\cos\theta}{\sqrt{2}}}^{\infty} e^{-s^2}ds$$

$$= \frac{1}{2\pi}e^{-\beta} + \sqrt{\frac{\beta}{4\pi}}\cos\theta\, e^{-\beta\sin^2\theta}[1 \pm \operatorname{erf}(\pm\sqrt{\beta}\cos\theta)]$$

where $\beta = S/N$

Thus,

$$\boxed{p_H(\theta_H) = p(-\theta_H) = \frac{1}{2\pi}e^{-\beta} + \sqrt{\frac{\beta}{4\pi}}\cos\theta_H\, e^{-\beta\sin^2\theta_H}[1 \pm \operatorname{erf}(\pm\sqrt{\beta}\cos\theta_H)];\ \cos\theta_H \gtrless 0}$$

Similarly,

$$\boxed{p_V(\theta_V) = \frac{1}{2\pi}e^{-\beta} + \sqrt{\frac{\beta}{4\pi}}\sin\theta_V\, e^{-\beta\cos^2\theta_V}[1 \pm \operatorname{erf}(\pm\sqrt{\beta}\sin\theta_V)];\ \sin\theta_V \gtrless 0}$$

Figures 4 and 5 show the single pulse probability densities for θ_H and θ_V for S/N = -3, 0, +3 dB.

With these single pulse density functions for θ_H and θ_V, we may obtain the density function for $\phi_{rel} = \theta_V - \theta_H$ as the convolution of $p_V(\theta_V)$ and $p_H(\theta_H)$ [1]. Neither the convolution nor the Fourier transforms of $p_V(\theta_V)$ and $p_H(\theta_H)$ appears feasible analytically, so we obtain $p_\phi(\phi_{rel})$ using characteristic functions and an FFT on the computer. Figure 6 shows the result of convolving the density functions of Figures 4 and 5.

The density function for $\sin\phi_{rel}$ is obtained using the relation

$$p_\phi(\phi)d\phi = p_Z(Z)dZ$$

where $Z = \sin\phi_{rel}$

Thus,

$$p_Z(Z) = \frac{p_\phi(\phi)}{\cos\phi}$$

Figure 7 shows the results when this transformation is applied to the densities of Figure 6.

For integration of N samples, the N-pulse density function is obtained by N convolutions of the single pulse density. Again we accomplish this using characteristic functions and FFT's on the computer. Figure 8 shows the result of integrating 4 pulses of the statistics in Figure 7. The threshold indicated is for a false alarm probability of 6.7×10^{-2}.

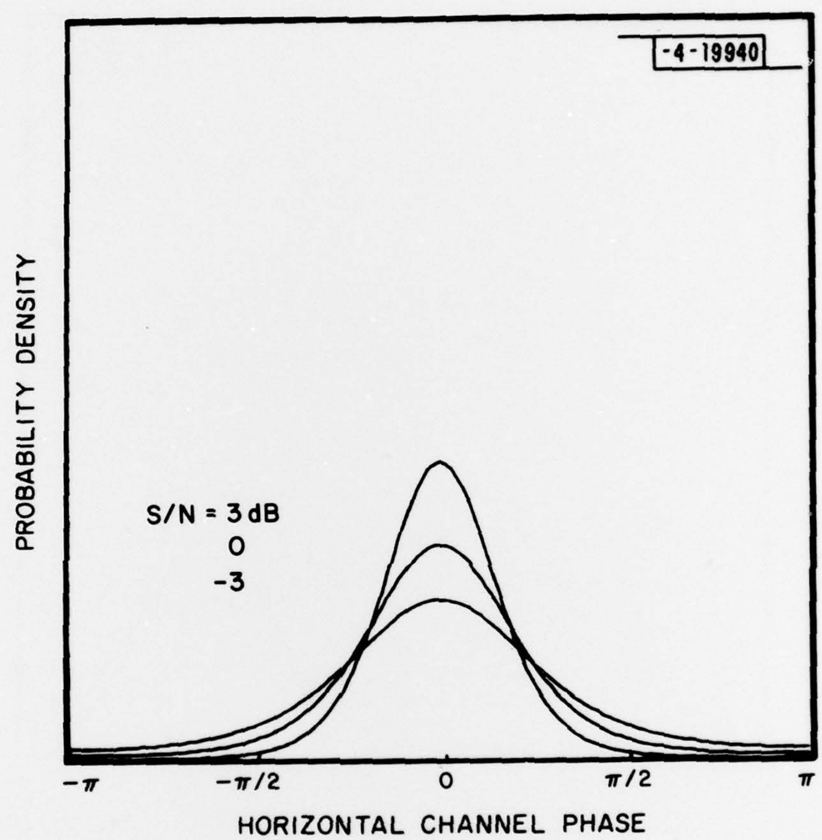

Fig. 4. Probability density for phase of horizontal channel, θ_H.

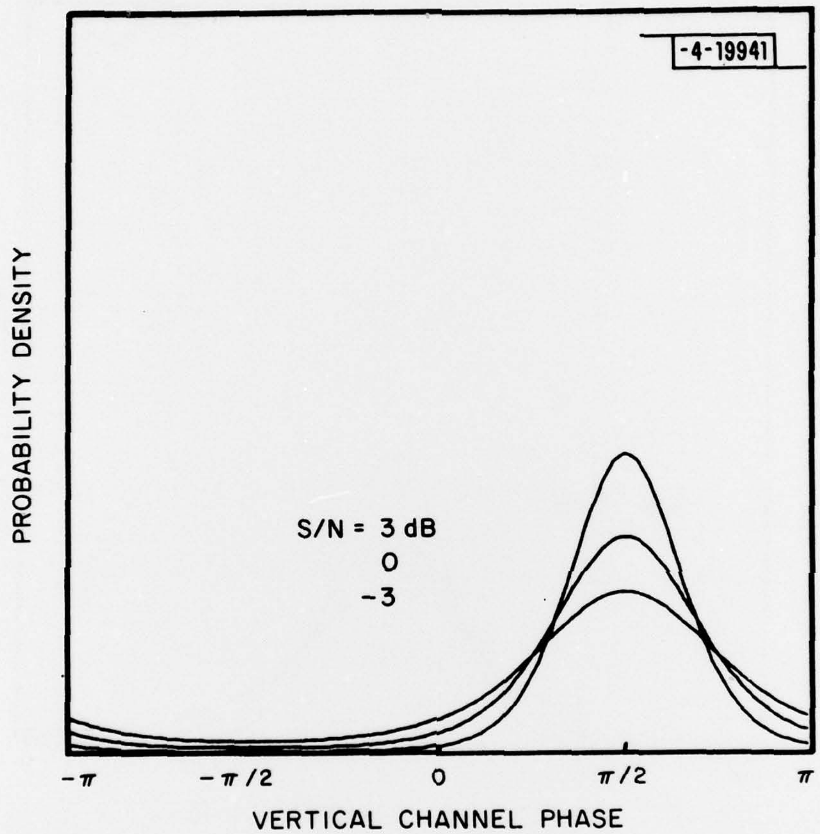

Fig. 5. Probability density for phase of vertical channel, θ_V.

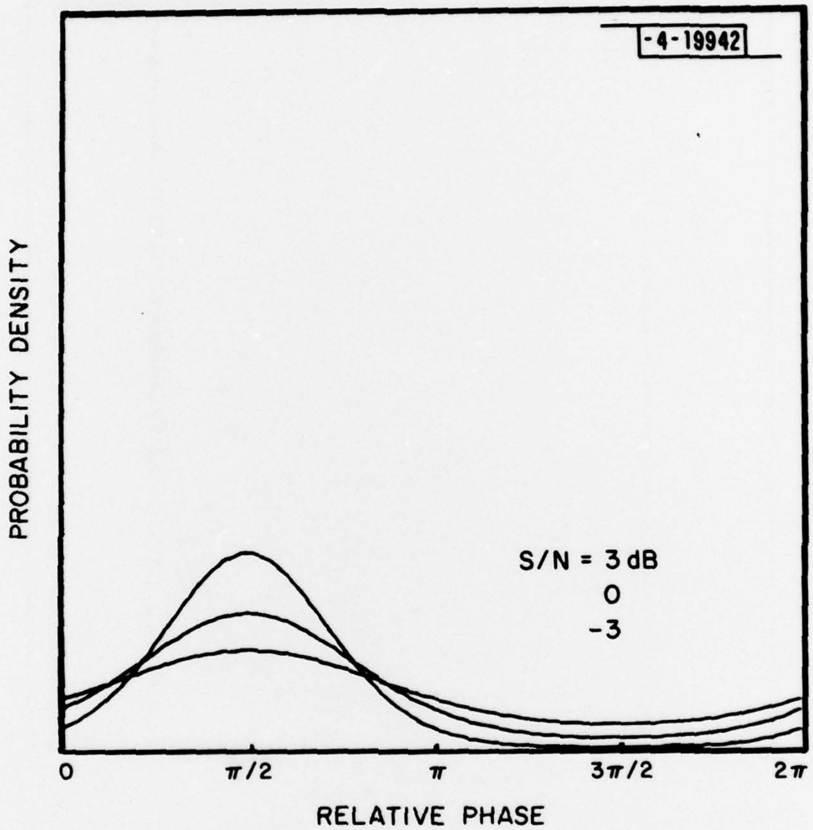

Fig. 6. Probability density for relative phase, $\phi_{rel} = \theta_V - \theta_H$.

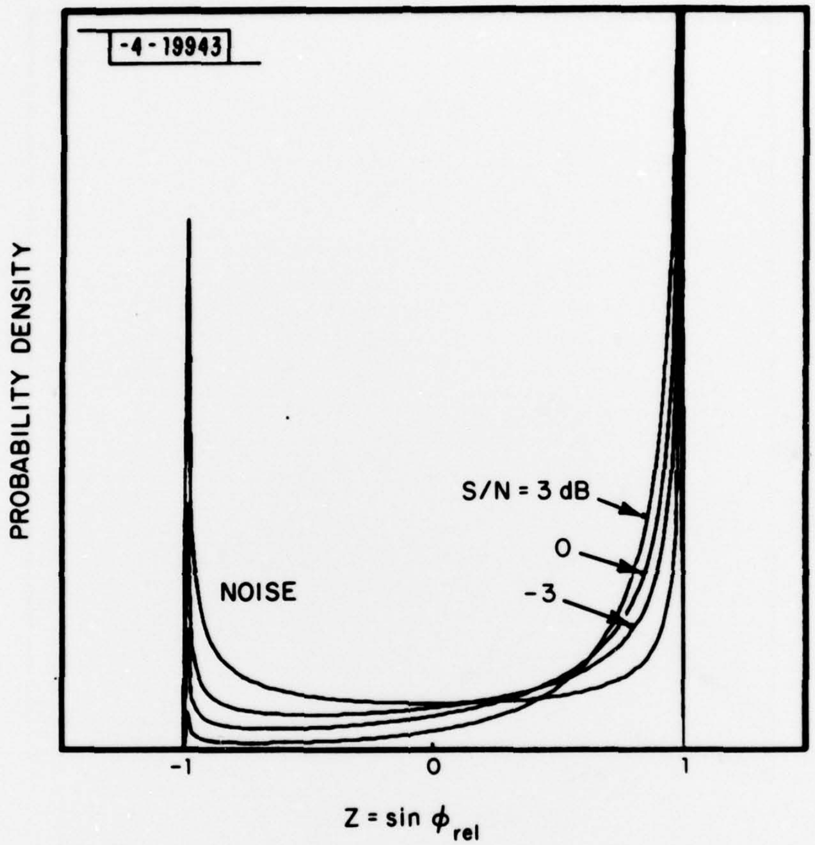

Fig. 7. Probability density for Z = sin ϕ_{rel}.

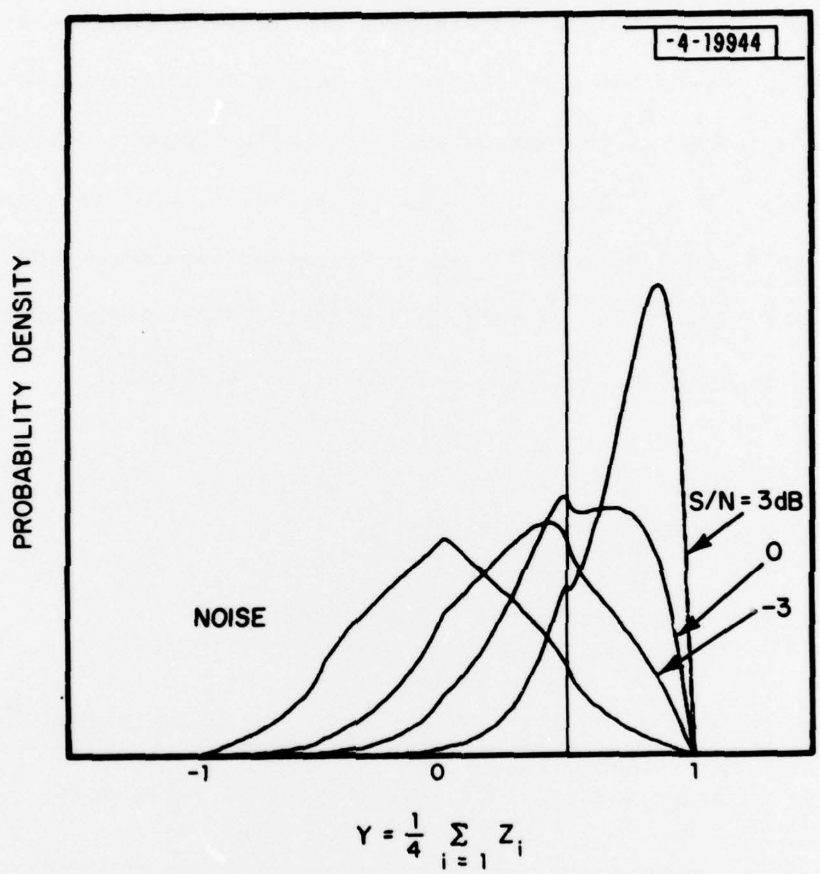

Fig. 8. 4-pulse detection problem: Probability densities for $Y = \frac{1}{4} \sum_{i=1}^{4} Z_i$ (noise only, S/N = −3, 0, 3 dB). Threshold set for 6.7×10^{-2} P_{FA}.

After obtaining the N-pulse statistics, a threshold may be set by numerically integrating the N-pulse noise only density to the right of the threshold.

The results of the above PCD analysis are shown in Figures 9-11 superimposed on the Marcum non-fluctuating target detection curves (taken from Fehlner's remake of the Marcum analysis [2]). Comparisons are for the number of pulses, N = 6, 10, and 30 and for probability of false alarm, $P_{fa} = 6.7 \times 10^{-2}$ and 6.93×10^{-4}. These P_{fa} selections were made to conform to Fehlner's use of false alarm number, n'. False alarm number and probability of false alarm are related by the expressions

$$(1 - P_{fa})^{n'} = 1/2$$

$$\text{or} \quad P_{fa} = 1 - 1/2^{1/n'}$$

Table I gives the conversion for n' used by Fehler.

TABLE 1

Relation Between False Alarm Number and False Alarm Probability

False Alarm Number n'	Probability of False Alarm P_{fa}
10^1	6.7×10^{-2}
10^3	6.93×10^{-4}
10^6	6.93×10^{-7}
10^8	6.93×10^{-9}
10^{10}	6.93×10^{-11}

For the three cases analyzed in Figures 9 - 11, it would appear that for high false alarm rates ($n' = 10^1$) PCD is equivalent to square law detection. Such is not the case, however, for the lower P_{fa}'s where the

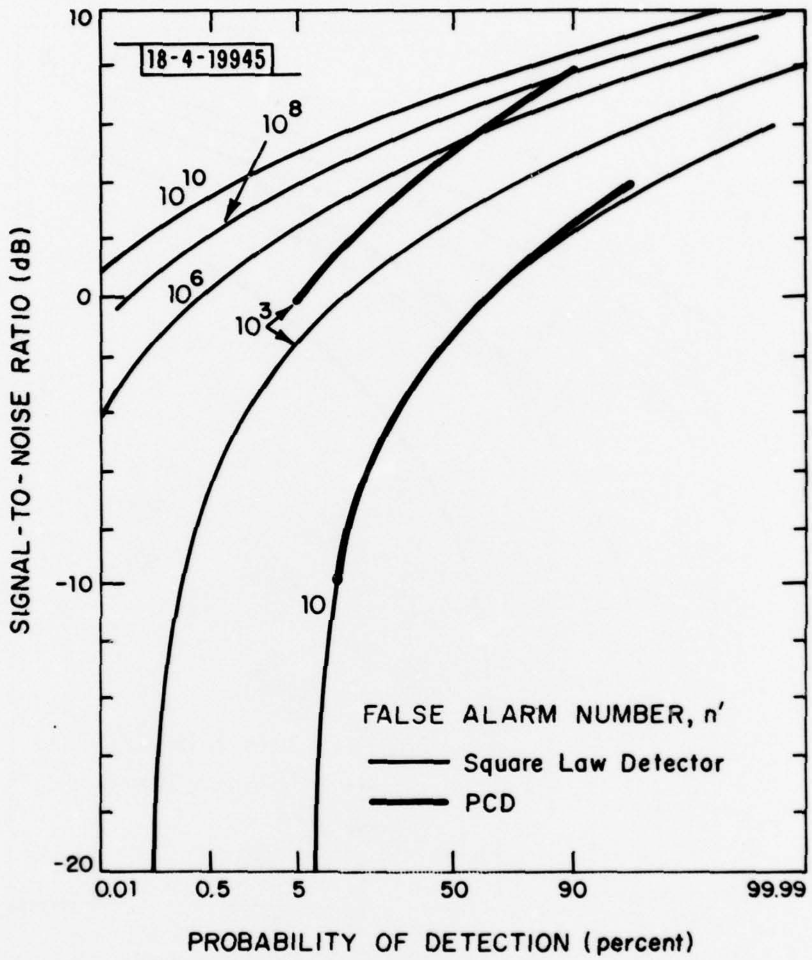

Fig. 9. Probability of detecting a non-fluctuating target. Pulses integrated, N = 6. Square law results taken from Fehlner [2].

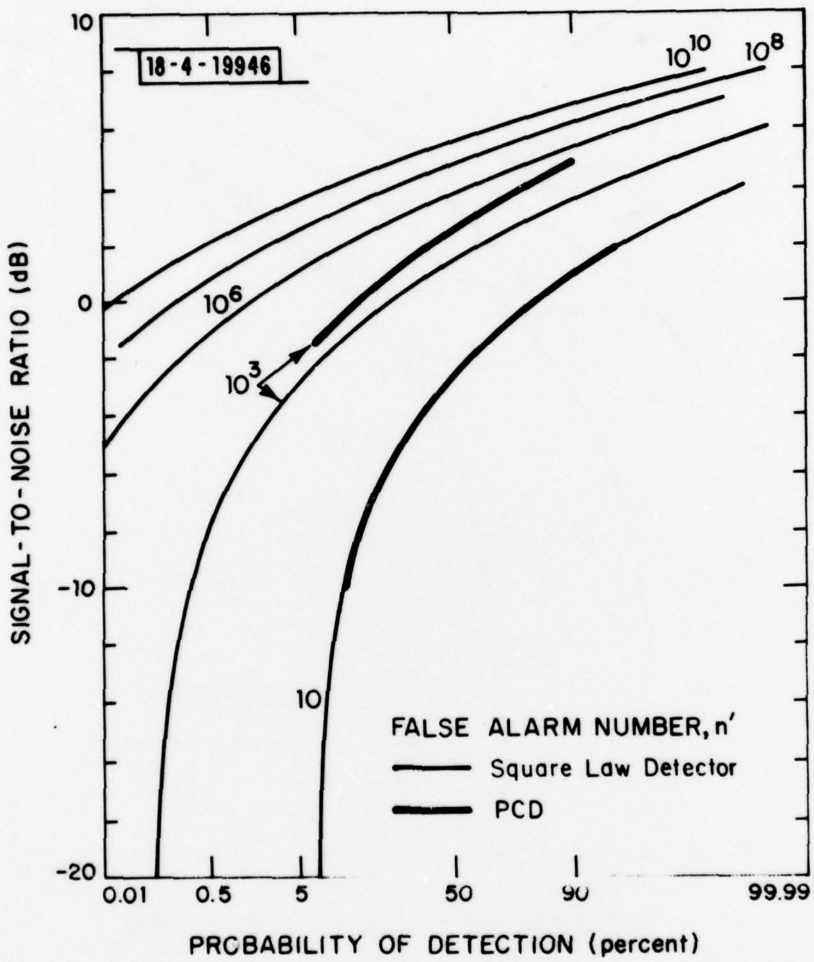

Fig. 10. Probability of detecting a non-fluctuating target. Pulses integrated, N = 10. Square law results taken from Fehlner [2].

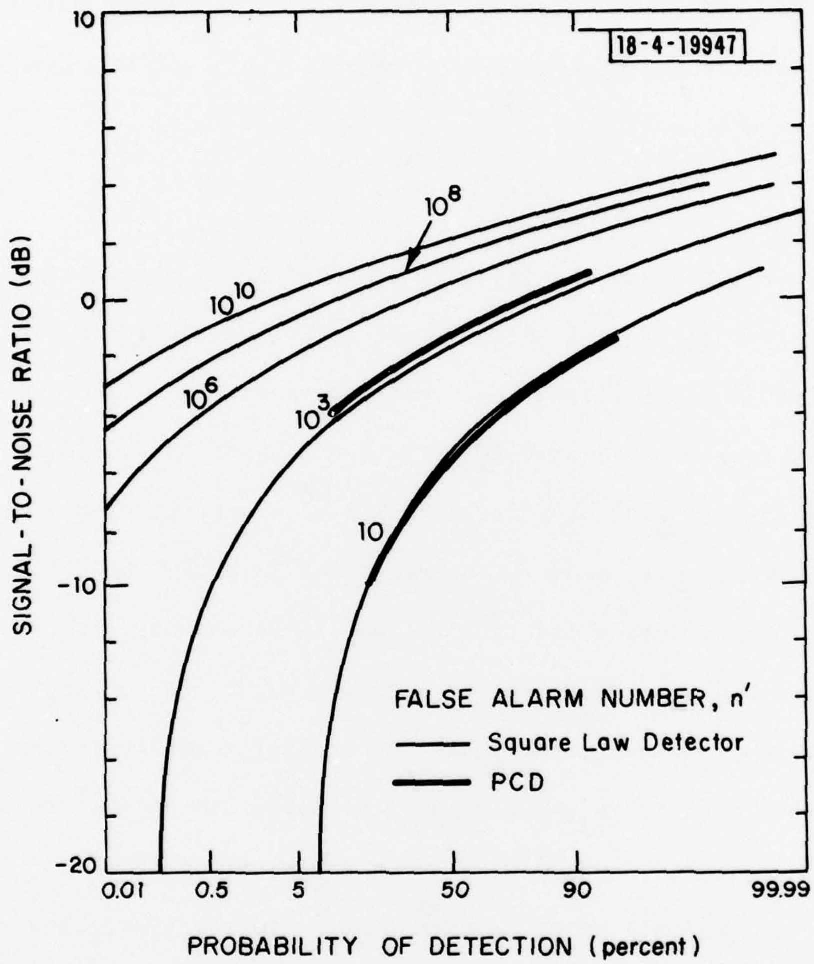

Fig. 11. Probability of detecting a non-fluctuating target. Pulses integrated, N = 30. Square law results taken from Fehlner [2].

PCD performance begins to degrade, although for larger N, this degradation is not as pronounced. It should be noted that for N = 30 in the PCD analysis, arithmetic underflow occurred so that Figure 11 may be somewhat questionable even though the underflow occurred in a non-critical segment of the program code.

OBSERVATIONS

Judging from the limited results presented here, it would appear that PCD does not perform as well as amplitude detection, at least for a steady target in noise. The performance is not substantially worse than amplitude detection, at least for relatively large N and might be considered (under the operating assumptions of this analysis) as a substitute processor of roughly equivalent performance. For detection in noise, of course, N is constrained only by system and application limits and may (in principle) be quite large, yielding substantial integration gain. For detection in clutter, however, the number of independent samples is bounded by the time-bandwidth product (in this application, the bandwidth is the frequency agile/non-coherent bandwidth), usually on the order of 30.

A rigorous analysis using more realistic clutter statistics probably will not affect the results substantially since phase and not amplitude is the discriminant and since integration of several pulses will drive the statistics toward Gaussian anyway. A better analysis should include a clutter model for which ϕ_{rel} is not uniformly distributed, but rather has some bias, since clutter may not completely depolarize the transmitted waveform. This will clearly degrade performance somewhat, but the extent is not clear. More importantly for the clutter model, is the allowance for clutter inhomogeneities. In this case, one may assume the relative phase is uniformly distributed from $-\pi$ to π on a cell-to-cell basis, but should not assume that the phase is so distributed on a frequency-to-frequency basis.

The target model is also of prime importance. The above analysis assumed the ideal case of a single perfect scatterer while more realistic target models should assume multiple scatterers. Multi-scatterer targets consisting only of even- or only of odd-bounce scatterers will interfere and change the S/C as the frequency is changed pulse-to-pulse, although maintaining the integrity of the relative phase between the vertical and horizontal receive channels. Multi-scatterer targets consisting of both types will interfere changing not only the S/C, but also the relative phase, and hence destroy the effectiveness of the technique altogether.

CONCLUSIONS

It is beyond the scope of this report to weigh all the advantages and disadvantages of this particular polarization discriminant, but a few observations can be made. Its principal advantage derives from the fact that to a first order, it is relatively independent of amplitude, eliminating the need for an adaptive threshold. Since it does not require information from adjacent cells for an adaptive threshold, implementation is simplified. Another advantage is that it does not require a coherent waveform.

The principal disadvantage established in this report is that the relative phase discriminant it extracts from two channels of information is at best the performance equal of square law detection using only a single channel. When placed on an equal basis, square law detection performance should improve as much as 3 dB when the amplitude information from both channels is used. Nor is hardware implementation of this technique merely a matter of adding another channel. The two channels must maintain relative phase coherence over the entire non-coherent bandwidth (usually between 200 and 500 MHz). The antenna design must also maintain this relative phase coherence as well as minimize cross-coupling effects between polarizations.

In any assessment of the proposed polarization technique, many trade-offs including those mentioned above must be considered. It seems clear, however, that in the absence of any unexpected clutter behavior, the polarization technique analyzed herein will not perform as well as classical amplitude detectors.

REFERENCES

[1] A. Papoulis, Probability, Random Variables, and Stochastic Processes. (McGraw-Hill, New York, 1965) p. 189.

[2] L. F. Fehlner, "Marcum's and Swerling's Data on Target Detection by a Pulsed Radar," Applied Physics Laboratory/Johns Hopkins University Report TG-451, Silver Spring, MD., July, 1962.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (*When Data Entered*)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS
BEFORE COMPLETING FORM

1. REPORT NUMBER
ESD-TR-78-395

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (*and Subtitle*)
A Gaussian Noise Analysis of the
"Pseudo-Coherent Discriminant"

5. TYPE OF REPORT & PERIOD COVERED
Technical Note

6. PERFORMING ORG. REPORT NUMBER
Technical Note 1978-46

7. AUTHOR(s)
Robert E. Stovall

8. CONTRACT OR GRANT NUMBER(s)
F19628-78-C-0002

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Lincoln Laboratory, M.I.T.
P.O. Box 73
Lexington, MA 02173

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Program Element No. 62702E
Project Code ~~9610~~
DARPA Order No. 2752

11. CONTROLLING OFFICE NAME AND ADDRESS
Defense Advanced Research Projects Agency
1400 Wilson Boulevard
Arlington, VA 22209

12. REPORT DATE
29 December 1978

13. NUMBER OF PAGES
26

14. MONITORING AGENCY NAME & ADDRESS (*if different from Controlling Office*)
Electronic Systems Division
Hanscom AFB
Bedford, MA 01731

15. SECURITY CLASS. (*of this report*)
Unclassified

15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (*of this Report*)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (*of the abstract entered in Block 20, if different from Report*)

18. SUPPLEMENTARY NOTES

None

19. KEY WORDS (*Continue on reverse side if necessary and identify by block number*)

radar detection
Gaussian noise
ground clutter
square law detection

20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

The "Pseudo-Coherent Discriminant" for radar detection of fixed targets in ground clutter is investigated. Performance of the technique in Gaussian noise interference is analyzed and compared with Marcum's results for square law detection of a steady signal.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (*When Data Entered*)